

UWThPh-1994-42
November 17, 1994

Isospin Violation in Chiral Perturbation Theory and the Decays $\eta \rightarrow \pi\ell\nu$ and $\tau \rightarrow \eta\pi\nu^*$

H. Neufeld
Institut für Theoretische Physik der Universität Wien
Boltzmanngasse 5, A-1090 Wien, Austria

A8241DAH at AWIUNI11.EDVZ.UNIVIE.AC.AT

Abstract

I discuss isospin breaking effects within the standard model. Chiral perturbation theory presents the appropriate theoretical framework for such an investigation in the low-energy range. Recent results on the electromagnetic contributions to the masses of the pseudoscalar mesons and the $K_{\ell 3}$ amplitudes are reported. Using the one-loop formulae for the $\eta_{\ell 3}$ form factors, rather precise predictions for the decay rates of $\eta \rightarrow \pi\ell\nu$ can be obtained. Finally, I present an estimate of the $\tau \rightarrow \eta\pi\nu$ branching ratio derived from the dominant meson resonance contributions to this decay.

To appear in the Proceedings of the 7th Adriatic Meeting in Particle Physics,
September 13-20, 1994, Brioni, Croatia

* Supported in part by Jubiläumsfonds der Oesterreichischen Nationalbank, Project No. 5051.

1 Introduction

In the standard model of strong and electroweak interactions the violation of the isospin symmetry has two different origins. First of all, it can be traced back to the different masses of up and down quarks. Besides that also electromagnetism induces isospin breaking effects.

In the confinement region of the standard model the usual perturbative methods are not applicable. In order to obtain testable theoretical prediction also in this case one has to resort to a so-called low-energy effective theory. With an appropriately chosen effective Lagrangian, chiral perturbation theory [1, 2, 3] (which is just the effective field theory of the standard model at low energies) is mathematically equivalent to the underlying fundamental theory [4]. Therefore, chiral perturbation theory presents the natural framework for the discussion of isospin breaking effects in the low energy range.

In this report I shall first sketch the construction of the effective chiral Lagrangian. I shall then show how electromagnetic interactions can be incorporated into the scheme of chiral perturbation theory. This machinery will then be applied to the mass formulae of the pseudoscalar mesons, in particular to the question of possible deviations from Dashen's limit [5]. Then I shall discuss the $K_{\ell 3}$ and $\eta_{\ell 3}$ form factors. The latter will then be used to obtain a standard model prediction for the decays $\eta \rightarrow \pi \ell \nu$ ($\ell = e, \mu$). Finally we shall leave the domain of chiral perturbation theory with an investigation of the decay $\tau \rightarrow \eta \pi \nu$.

This talk is based on a recent paper [6] by H. Rupertsberger and myself to which the reader is referred for a more detailed discussion.

2 The Low-Energy Limit of QCD

The guiding principle for the construction of the effective low-energy theory of QCD is the spontaneously broken chiral group $SU(3)_L \times SU(3)_R$. The Goldstone bosons of chiral symmetry breaking serve as the asymptotic fields of the low-energy effective theory. With these basic building blocks one constructs the most general relativistic quantum field theory respecting the chiral as well as the discrete symmetries P and C.

To lowest order in the chiral expansion, the effective Lagrangian is given by

$$\mathcal{L}_2 = \frac{F^2}{4} \langle D_\mu U D^\mu U^\dagger + \chi U^\dagger + \chi^\dagger U \rangle, \quad (1)$$

where

$$D_\mu U = \partial_\mu U - i(v_\mu + a_\mu)U + iU(v_\mu - a_\mu), \quad \chi = 2B(s + ip), \quad (2)$$

and $\langle \dots \rangle$ denotes the trace in three-dimensional flavour space. U is a unitary and unimodular 3×3 matrix which transforms as

$$U \rightarrow g_R U g_L^\dagger, \quad g_{L,R} \in SU(3)_{L,R}, \quad (3)$$

under the chiral group. It incorporates the fields of the eight pseudoscalar mesons. The fields v_μ, a_μ, s, p are vector, axial vector, scalar and pseudoscalar external source terms, respectively. The quark mass matrix

$$\mathcal{M} = \text{diag}(m_u, m_d, m_s) \quad (4)$$

is contained in the scalar field $s(x)$ which incorporates with $m_u \neq m_d$ already the first source of isospin violation. The parameters F and B are the only free constants of \mathcal{L}_2 : F is the pion decay

constant in the chiral limit, whereas B is related to the quark condensate. The Lagrangian in (1) is referred to as the effective chiral Lagrangian of $\mathcal{O}(p^2)$. The chiral counting rules are the following: The field U is of $\mathcal{O}(p^0)$, the derivative ∂_μ and the external gauge fields v_μ, a_μ are terms of $\mathcal{O}(p)$ and the fields s, p count as $\mathcal{O}(p^2)$.

At the next to leading $\mathcal{O}(p^4)$ the one-loop functional generated by the lowest order Lagrangian (1) is renormalized by the Gasser–Leutwyler Lagrangian [3]

$$\mathcal{L}_4 = \sum_{i=1}^{12} L_i P_i. \quad (5)$$

The P_i are operators of $\mathcal{O}(p^4)$ like $P_1 = \langle D_\mu U D^\mu U^\dagger \rangle^2$, etc. The finite parts of the ten low-energy constants L_1, \dots, L_{10} can be determined [2, 3] by using experimental input. (P_{11} and P_{12} are contact terms which are not directly accessible to experiment.)

3 Electromagnetic Interactions

The effective Lagrangian discussed so far allows also the description of electromagnetic processes, provided that the electromagnetic gauge potential $A_\mu(x)$ can be treated as an external field. As we are interested in isospin breaking effects induced by electromagnetism, such an approach is not sufficient anymore. In this case, the photon field must be included as an additional dynamical degree of freedom. One-photon loops will now contribute corrections of order e^2 .

The corresponding local counterterms can be constructed by introducing [7] spurion fields $Q_{L,R}(x)$ which transform under the chiral group as

$$Q_L \rightarrow g_L Q_L g_L^\dagger, \quad Q_R \rightarrow g_R Q_R g_R^\dagger. \quad (6)$$

At the end, one identifies $Q_{L,R}$ with the quark charge matrix Q . The covariant derivatives are given by

$$D_\mu Q_L = \partial_\mu Q_L - i[v_\mu - a_\mu, Q_L], \quad D_\mu Q_R = \partial_\mu Q_R - i[v_\mu + a_\mu, Q_R]. \quad (7)$$

To lowest $\mathcal{O}(e^2 p^0)$ the electromagnetic effective Lagrangian contains a single term

$$\mathcal{L}|_{\mathcal{O}(e^2 p^0)} = F^4 e^2 Z \langle Q U^\dagger Q U \rangle, \quad (8)$$

with a real and dimensionless coupling constant Z . The effective Lagrangians (1) and (8) contain the lowest-order contributions to the masses of the pseudoscalar mesons from QCD and the electromagnetic interaction, respectively:

$$\begin{aligned} \widehat{M}_{\pi^\pm}^2 &= 2B\widehat{m} + 2e^2 Z F^2, \\ \widehat{M}_{\pi^0}^2 &= 2B\widehat{m}, \\ \widehat{M}_{K^\pm}^2 &= B \left[(m_s + \widehat{m}) - \frac{2\varepsilon}{\sqrt{3}}(m_s - \widehat{m}) \right] + 2e^2 Z F^2, \\ \widehat{M}_{K^0}^2 &= B \left[(m_s + \widehat{m}) + \frac{2\varepsilon}{\sqrt{3}}(m_s - \widehat{m}) \right], \\ \widehat{M}_\eta^2 &= \frac{4}{3}B \left(m_s + \frac{\widehat{m}}{2} \right), \end{aligned} \quad (9)$$

where \hat{m} denotes the mean value of the light quark masses,

$$\hat{m} = \frac{1}{2}(m_u + m_d). \quad (10)$$

The mixing angle

$$\varepsilon = \frac{\sqrt{3}}{4} \frac{m_d - m_u}{m_s - \hat{m}} \quad (11)$$

relates π_3, π_8 to the (tree-level) mass eigenfields $\hat{\pi}_0, \hat{\eta}$:

$$\begin{aligned} \pi_3 &= \hat{\pi}_0^0 - \varepsilon \hat{\eta}, \\ \pi_8 &= \varepsilon \hat{\pi}_0^0 + \hat{\eta}. \end{aligned} \quad (12)$$

Terms of higher than linear order in ε have been neglected. In accordance with Dashen's theorem [5], the lowest order electromagnetic Lagrangian (8) contributes an equal amount to the squared masses of π^\pm, K^\pm . It does not contribute to the masses of π^0, K^0, \bar{K}^0 or η , nor does it generate π^0 - η mixing.

At next-to-leading $\mathcal{O}(e^2 p^2)$ one finds the following list of local counterterms [6, 8]

$$\begin{aligned} \mathcal{L}|_{\mathcal{O}(e^2 p^2)} &= F^2 e^2 \{ K_1 \langle Q^2 \rangle \langle D_\mu U D^\mu U^\dagger \rangle + K_2 \langle U Q U^\dagger Q \rangle \langle D_\mu U D^\mu U^\dagger \rangle \\ &\quad + K_3 [\langle Q U^\dagger D_\mu U \rangle \langle Q U^\dagger D^\mu U \rangle + \langle Q U D_\mu U^\dagger \rangle \langle Q U D^\mu U^\dagger \rangle] \\ &\quad + K_4 \langle D_\mu U Q U^\dagger \rangle \langle D^\mu U^\dagger Q U \rangle + K_5 \langle Q^2 (D_\mu U^\dagger D^\mu U + D_\mu U D^\mu U^\dagger) \rangle \\ &\quad + K_6 \langle U Q U^\dagger Q D_\mu U D^\mu U^\dagger + Q U Q U^\dagger D_\mu U D^\mu U^\dagger \rangle \\ &\quad + K_7 \langle Q^2 \rangle \langle \chi U^\dagger + \chi^\dagger U \rangle + K_8 \langle U Q U^\dagger Q \rangle \langle \chi U^\dagger + \chi^\dagger U \rangle \\ &\quad + K_9 \langle Q^2 (U^\dagger \chi + \chi^\dagger U + \chi U^\dagger + U \chi^\dagger) \rangle \\ &\quad + K_{10} \langle Q U^\dagger Q \chi + U Q U^\dagger Q U \chi^\dagger + U^\dagger Q U Q U^\dagger \chi + Q U Q \chi^\dagger \rangle \\ &\quad - K_{11} \langle Q U^\dagger Q \chi - U Q U^\dagger Q U \chi^\dagger - U^\dagger Q U Q U^\dagger \chi + Q U Q \chi^\dagger \rangle \\ &\quad + K_{12} \langle D_\mu U^\dagger [D^\mu Q_R, Q_R] U + D_\mu U [D^\mu Q_L, Q_L] U^\dagger \rangle \\ &\quad + K_{13} \langle U D_\mu Q_L U^\dagger D^\mu Q_R \rangle + K_{14} \langle D_\mu Q_L D^\mu Q_L + D_\mu Q_R D^\mu Q_R \rangle \}. \end{aligned} \quad (13)$$

As in the strong sector, the divergences of the electromagnetic parts of the one-loop graphs are absorbed by an appropriate renormalization [8] of the coupling constants K_i .

If the class of observables is restricted to the masses of the pseudoscalar mesons, the decay constants F_P and the $P_{\ell 3}$ form factors, we may confine ourselves to the following eight linear combinations of the electromagnetic coupling constants [6]:

$$\begin{aligned} S_1 &= K_1 + K_2, & S_2 &= K_5 + K_6, & S_3 &= -2K_3 + K_4, \\ S_4 &= K_7 + K_8, & S_5 &= K_9 + 2K_{10} + K_{11}, & S_6 &= K_8, \\ S_7 &= K_{10} + K_{11}, & S_8 &= -K_{12}. \end{aligned} \quad (14)$$

4 Pseudoscalar Masses

As an illustrative example for the application of chiral perturbation theory let me discuss some observables built out of the masses of the pseudoscalar mesons.

The difference of the squared pion masses can be used for extracting some information about the electromagnetic interaction. Up to tiny corrections of $\mathcal{O}(\varepsilon^2)$, this observable is purely electromagnetic,

$$M_{\pi^\pm}^2 - M_{\pi^0}^2 = 2e^2 Z F^2 + 2e^2 M_K^2 \left[4S_6^r(\mu) - 16Z L_4^r(\mu) - \frac{Z}{(4\pi)^2} \ln \frac{M_K^2}{\mu^2} \right] + \mathcal{O}(e^2 M_\pi^2). \quad (15)$$

This formula exhibits the typical structure of a one-loop result in chiral perturbation theory. It contains a so-called chiral logarithm and, in addition, contributions from local counterterms. The scale dependence [3, 6] of the renormalized low-energy constants is such that the total (observable) result is independent of the scale parameter μ . At this point we are confronted with the problem that the numerical values of the electromagnetic coupling constants $S_i^r(\mu)$ are unknown. Chiral dimensional analysis only suggests the approximate upper bound

$$|S_i^r(M_\rho)| \lesssim \frac{1}{(4\pi)^2} = 6.3 \cdot 10^{-3}. \quad (16)$$

The assumption that the experimental value of $M_{\pi^\pm}^2 - M_{\pi^0}^2$ is largely dominated by the leading term of $\mathcal{O}(e^2 p^0)$,

$$(M_{\pi^\pm}^2 - M_{\pi^0}^2)_{\text{exp}} \simeq 2e^2 Z F^2, \quad (17)$$

together with [9] $F = 92.4$ MeV and [3] $L_4^r(M_\rho) = (-0.3 \pm 0.5) \cdot 10^{-3}$ would imply

$$S_6^r(M_\rho) \approx (-2.1 \pm 1.6) \cdot 10^{-3}, \quad (18)$$

which is in accordance with (16). Let me remark that the validity of (17) is also supported by theoretical arguments [7, 10] based on resonance exchange.

A further relation which is sensitive to electromagnetic contributions is given by

$$\begin{aligned} \frac{m_d^2 - m_u^2}{m_s^2 - \hat{m}^2} &= \left[(M_{K^0}^2 - M_{K^\pm}^2 + M_{\pi^\pm}^2 - M_{\pi^0}^2)_{\text{exp}} - (M_{K^0}^2 - M_{K^\pm}^2 + M_{\pi^\pm}^2 - M_{\pi^0}^2)_{\text{EM}} \right] \\ &\quad \cdot \frac{M_\pi^2}{(M_K^2 - M_\pi^2) M_K^2}, \end{aligned} \quad (19)$$

where [6, 8]

$$\begin{aligned} (M_{K^0}^2 - M_{K^\pm}^2 + M_{\pi^\pm}^2 - M_{\pi^0}^2)_{\text{EM}} &= e^2 M_K^2 \left[\frac{1}{(4\pi)^2} \left(3 \ln \frac{M_K^2}{\mu^2} - 4 + 2Z \ln \frac{M_K^2}{\mu^2} \right) \right. \\ &\quad \left. + \frac{4}{3} S_2^r(\mu) - 8S_7^r(\mu) + 16Z L_5^r(\mu) \right] + \mathcal{O}(e^2 M_\pi^2). \end{aligned} \quad (20)$$

It is obvious that (19) provides us with an important information about the quark mass ratio

$$1/Q^2 := \frac{m_d^2 - m_u^2}{m_s^2 - \hat{m}^2}. \quad (21)$$

The only uncertainty in the determination of (21) is again the unknown electromagnetic low-energy constant appearing in (20). In Dashen's limit (corresponding to a vanishing electromagnetic contribution), the ratio of quark masses (21) is given by $1/Q^2 = 1.72 \cdot 10^{-3}$. This value would be reproduced for

$$S_2^r(M_\rho) - 6S_7^r(M_\rho) = (24.8 \pm 4.8) \cdot 10^{-3}. \quad (22)$$

In deriving (22) I have used [3] $L_5^r(M_\rho) = (1.4 \pm 0.5) \cdot 10^{-3}$ and $Z = 0.8$ taken from (17). The number in (22) suggests again that the individual coupling constants $S_i^r(M_\rho)$ do not exceed the rather generous order of magnitude estimate (16). Nevertheless, because of the large coefficients occurring in (20), sizeable deviations from the Dashen limit cannot be excluded.

As an illustration, let us vary the electromagnetic coupling constant in (20) within the range

$$-\frac{7}{(4\pi)^2} \leq S_2^r(M_\rho) - 6S_7^r(M_\rho) \leq \frac{7}{(4\pi)^2}. \quad (23)$$

From (23) one obtains

$$1.5 \cdot 10^{-3} \lesssim 1/Q^2 = \frac{m_d^2 - m_u^2}{m_s^2 - \hat{m}^2} \lesssim 2.4 \cdot 10^{-3}. \quad (24)$$

The quantity Q appears also in the analysis of $\eta \rightarrow 3\pi$ decays. The calculation [11] of these reactions has been performed at the one-loop level in chiral perturbation theory. As the corresponding amplitudes are proportional to $1/Q^2$, the current experimental data [9] can be used to extract a value for Q . However, the corrections of $\mathcal{O}(p^4)$ increase the current algebra value for the transition amplitude by roughly 50%, whereas the electromagnetic contributions are effectively of $\mathcal{O}(e^2 m_{u,d})$ and thus expected to be small [11]. The unusually large correction of $\mathcal{O}(p^4)$ is due to strong final state interaction effects. As a consequence, the numerical accuracy of the theoretical prediction is rather modest [12]. If one is nevertheless willing to use the one-loop result for $\eta \rightarrow 3\pi$, the present experimental data are favouring a quark mass ratio around $1/Q^2 = 2.3 \cdot 10^{-3}$. Values of approximately this size are also supported by certain model calculations [13, 14].

The knowledge of the size of the parameter Q provides us with an important information for the determination of the quark mass ratios m_u/m_d and m_s/m_d . This can be seen most easily by writing (21) in the form of Leutwyler's ellipse [15],

$$\left(\frac{m_u}{m_d}\right)^2 + \frac{1}{Q^2} \left(\frac{m_s}{m_d}\right)^2 = 1. \quad (25)$$

One might now worry that the large uncertainties in the determination of Q due to electromagnetism could call in question the standard results [15] for the quark mass ratios. However, there are still additional constraints [15] on m_u/m_d and m_s/m_d from the mass splitting of the baryons [16, 17] and from an analysis of $\eta - \eta'$ mixing [3]. The whole situation is displayed in Fig 1. We see that in the relevant region (determined by the restrictions from R and the $\eta - \eta'$ mixing angle) the effect of a possibly large deviation of Q from Dashen's value is not too dramatic.

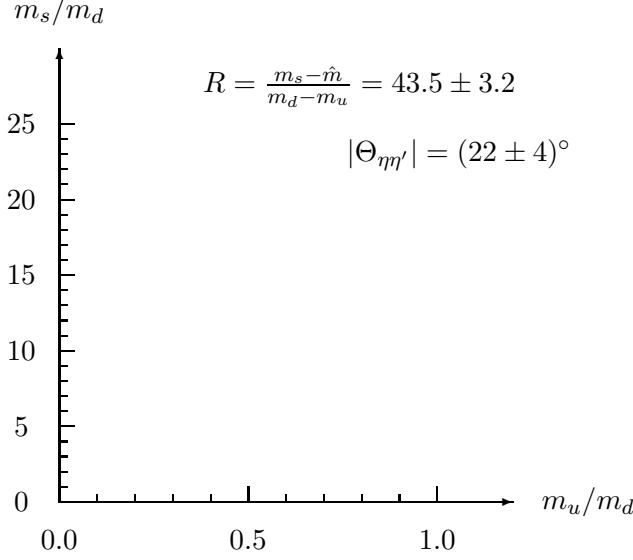


Figure 1: The elliptic band corresponds to the range (24).

5 $K_{\ell 3}$ form factors

After having seen the potentially large electromagnetic contributions in the spectrum of the pseudoscalars one might expect the same feature also for other isospin violating observables. The following example will show you that this is in general not the case. For the ratio $r_{K\pi}$ of the $K_{\ell 3}$ form factors one finds the following expression [6]:

$$r_{K\pi} = \frac{f_+^{K^+\pi^0}(0)}{f_+^{K^0\pi^-}(0)} = 1 + \sqrt{3} \left(\varepsilon - \frac{M_{\hat{\pi}^0\hat{\eta}}^2}{M_\eta^2 - M_\pi^2} \right) + \frac{3e^2}{4(4\pi)^2} \ln \frac{M_K^2}{M_\pi^2}. \quad (26)$$

$M_{\hat{\pi}^0\hat{\eta}}^2$ is the off-diagonal element of the $\pi^0 - \eta$ mass matrix in the basis of the tree-level mass eigenfields $\hat{\pi}^0, \hat{\eta}$. Explicitly, it is given by [6]

$$\begin{aligned} M_{\hat{\pi}^0\hat{\eta}}^2 = & \frac{2\varepsilon}{3(4\pi)^2 F^2} \left\{ 64(4\pi)^2 (M_K^2 - M_\pi^2)^2 [3L_7 + L_8^r(\mu)] \right. \\ & - M_\eta^2 (M_K^2 - M_\pi^2) \ln \frac{M_\eta^2}{\mu^2} - 2M_K^2 (M_K^2 - 2M_\pi^2) \ln \frac{M_K^2}{\mu^2} \\ & \left. + M_\pi^2 (M_K^2 - 3M_\pi^2) \ln \frac{M_\pi^2}{\mu^2} - 2M_K^2 (M_K^2 - M_\pi^2) \right\} \\ & - \frac{2\sqrt{3}e^2 M_K^2}{27(4\pi)^2} \left\{ 2(4\pi)^2 [2S_2^r(\mu) + 3S_3^r(\mu)] - 9Z \left(\ln \frac{M_K^2}{\mu^2} - 1 \right) \right\}. \end{aligned} \quad (27)$$

In the limit $e = 0$, the formula for $r_{K\pi}$ has already been worked out [18, 19] some time ago. The data on the decays $K^+ \rightarrow \pi^0 e^+ \nu_e$ and $K^0 \rightarrow \pi^- e^+ \nu_e$ show clear evidence for the presence

of isospin breaking. Dividing the rates by the relevant phase space integrals (including those electromagnetic corrections which are sensitive to the lepton kinematics [18]) one finds [19]

$$\left| \frac{f_+^{K^+\pi^0}(0)}{f_+^{K^0\pi^-}(0)} \right|^2 = 1.057 \pm 0.019, \quad (28)$$

which implies

$$(r_{K\pi} - 1)_{\text{exp}} = (2.8 \pm 0.9) \cdot 10^{-2}. \quad (29)$$

For our numerical analysis [6] we have used the value for ε extracted from the mass splitting in the baryon octet [16, 17],

$$\varepsilon = (1.00 \pm 0.07) \cdot 10^{-2}. \quad (30)$$

With the usual numerical values [3] for the low-energy constants L_7, L_8^T , the QCD contribution to $r_{K\pi} - 1$ is given by

$$(r_{K\pi} - 1)_{\text{QCD}} = 2.1 \cdot 10^{-2}. \quad (31)$$

Assuming the validity of (16), the electromagnetic contributions are expected to be rather small,

$$0 \lesssim (r_{K\pi} - 1)_{\text{EM}} \lesssim 0.2 \cdot 10^{-2}. \quad (32)$$

6 $\eta \rightarrow \pi \ell \nu$

Also the $\eta_{\ell 3}$ form factors $f_{\pm}^{\eta\pi}(t)$ have been calculated [6] at the one-loop level in chiral perturbation theory including the electromagnetic contributions of $\mathcal{O}(e^2 p^2)$. Quite remarkably, $f_+^{\eta\pi}(0)$ can be related to the ratio of $K_{\ell 3}$ form factors $r_{K\pi}$ by the parameter free relation

$$f_+^{\eta\pi}(0) = \frac{1}{\sqrt{3}} \left[r_{K\pi} - 1 - \frac{3e^2}{4(4\pi)^2} \ln \frac{M_K^2}{M_\pi^2} \right]. \quad (33)$$

Inserting the experimental value for $r_{K\pi} - 1$ given in (29), we obtain the prediction

$$f_+^{\eta\pi}(0) = (1.6 \pm 0.5) \cdot 10^{-2}. \quad (34)$$

Although interesting in principle, this approach is drastically depreciated by the large error in (34). The more promising strategy is certainly to employ again the input parameters which have already been used in deriving (31) and (32). In this case, we obtain

$$f_+^{\eta\pi}(0)|_{\text{QCD}} = 1.21 \cdot 10^{-2} \quad (35)$$

for the QCD contributions. As in the $K_{\ell 3}$ case the electromagnetic contributions are expected to be rather small,

$$0.02 \cdot 10^{-2} \lesssim f_+^{\eta\pi}(0)|_{\text{EM}} \lesssim 0.15 \cdot 10^{-2}. \quad (36)$$

With the full expressions for the form factors $f_{\pm}^{\eta\pi}(t)$ the branching ratios of the decays $\eta \rightarrow \pi \ell \nu$ ($\ell = e, \mu$) can be computed [6]. Taking into account the uncertainties due to electromagnetic contributions which have been estimated by using (16), one obtains [6]

$$\begin{aligned} 4.7 \cdot 10^{-14} &\lesssim BR(\eta \rightarrow \pi^+ e^- \bar{\nu}_e) \lesssim 5.8 \cdot 10^{-14}, \\ 3.4 \cdot 10^{-14} &\lesssim BR(\eta \rightarrow \pi^+ \mu^- \bar{\nu}_\mu) \lesssim 4.1 \cdot 10^{-14}. \end{aligned} \quad (37)$$

Adding all four decay channels, one arrives at [6]

$$1.6 \cdot 10^{-13} \lesssim \sum_{\ell=e,\mu} BR(\eta \rightarrow \pi^\pm \ell^\mp \begin{smallmatrix} (-) \\ \nu_\ell \end{smallmatrix}) \lesssim 2.0 \cdot 10^{-13}. \quad (38)$$

The most powerful source of η particles is presently installed at the SATURNE synchrotron in Saclay. In this η factory, the reaction $pd \rightarrow {}^3\text{He}\eta$ serves as a source of $\sim 10^8$ tagged η particles per day [20]. So we see that the reaction $\eta \rightarrow \pi\ell\nu$ is still out of the range of present experimental facilities. On the other hand, the observation of a decay rate considerably larger than the upper bound in (38) would be a clear signal for a deviation from the standard model.

7 $\tau \rightarrow \eta\pi\nu$

The decay $\tau \rightarrow \eta\pi\nu$ is sensitive to the same hadronic form factors as $\eta \rightarrow \pi\ell\nu$. However, for the τ decay the invariant mass of the $\eta\pi$ system lies in the range $M_\eta + M_\pi \leq \sqrt{t} \leq m_\tau$, which is already outside the domain of applicability of chiral perturbation theory. In order to obtain reasonable theoretical results also in this intermediate energy range the dominant contributions of the lowest-lying resonance states have to be taken into account [6]. The presence of these resonances shows up by the appearance of poles in the amplitudes for certain values of the kinematical variable t .

Meson resonances can be incorporated in the effective chiral Lagrangian as additional degrees of freedom [7, 21]. They carry nonlinear realizations of the chiral group G depending on their transformation properties under the diagonal subgroup $SU(3)_V$.

In the case of $\tau \rightarrow \eta\pi\nu$ one has to consider the resonance contributions from $\rho(770)$ and $a_0(980)$. The final result [6] for the hadronic form factors contains the vector decay constant F_{a_0} as the only free parameter. Taking $|F_{a_0}| = 1.28 \text{ MeV}$ from a QCD sum rule analysis [22], we obtained the prediction [6] $BR(\tau \rightarrow \eta\pi\nu) \simeq 1.2 \cdot 10^{-5}$, to be compared with the present experimental bound [23] $BR(\tau \rightarrow \eta\pi\nu)|_{\text{exp}} < 3.4 \cdot 10^{-4}$. Therefore, the detection of the decay $\tau \rightarrow \eta\pi\nu$ is to be expected in the near future. The measurement of the decay rate can then be used for a determination of F_{a_0} .

Acknowledgements

I wish to thank the organizers for the pleasant and stimulating atmosphere of this conference and G. Ecker for reading the manuscript.

References

References

- [1] S. Weinberg, *Physica* 96A (1979) 327.
- [2] J. Gasser and H. Leutwyler, *Ann. Phys.* 158 (1984) 142.
- [3] J. Gasser and H. Leutwyler, *Nucl. Phys.* B250 (1985) 465.
- [4] H. Leutwyler, *Ann. Phys.* 235 (1994) 165.
- [5] R. Dashen, *Phys. Rev.* 183 (1969) 1245.
- [6] H. Neufeld and H. Rupertsberger, *Isospin Breaking in Chiral Perturbation Theory and the Decays $\eta \rightarrow \pi\ell\nu$ and $\tau \rightarrow \eta\pi\nu$* , University of Vienna preprint UWThPh-1994-15, submitted for publication in *Nucl. Phys. B*.
- [7] G. Ecker, J. Gasser, A. Pich and E. de Rafael, *Nucl. Phys.* B321 (1989) 311.
- [8] A. Urech, *Virtual Photons in Chiral Perturbation Theory*, University of Berne preprint BUTP-94/9 (1994).
- [9] Particle Data Group, *Phys. Rev. D* 50 (1994) Number 3, Part I.
- [10] T. Das et al., *Phys. Rev. Lett.* 18 (1967) 759.
- [11] J. Gasser and H. Leutwyler, *Nucl. Phys.* B250 (1985) 539.
- [12] H. Leutwyler, *Masses of the light quarks*, University of Berne preprint BUTP-94/8 (1994).
- [13] J.F. Donoghue, B.R. Holstein and D. Wyler, *Phys. Rev. D* 47 (1993) 2089.
- [14] J. Bijnens, *Phys. Lett.* B306 (1993) 343.
- [15] H. Leutwyler, *Nucl. Phys.* B337 (1990) 108.
- [16] J. Gasser, *Ann. Phys.* 136 (1981) 62.
- [17] J. Gasser and H. Leutwyler, *Phys. Rep.* 87 (1982) 77.
- [18] H. Leutwyler and M. Roos, *Z. Phys.* C25 (1984) 91.
- [19] J. Gasser and H. Leutwyler, *Nucl. Phys.* B250 (1985) 517.
- [20] R.S. Kessler et al., *Phys. Rev. Lett.* 70 (1993) 892.
- [21] G. Ecker, J. Gasser, H. Leutwyler, A. Pich and E. de Rafael, *Phys. Lett.* B223 (1989) 425.
- [22] S. Narison, *Rivista del Nuovo Cimento*, vol. 10, no. 2 (1987) 1.
- [23] M. Artuso et al., *Phys. Rev. Lett.* 69 (1992) 3278.

This figure "fig1-1.png" is available in "png" format from:

<http://arXiv.org/ps/hep-ph/9412207v1>